Continuous Time Equations for Analog Tape Modeling

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Hysteresis

The magnetostatic field recorded to magnetic tape can be described using a hysteresis loop. A circuit simulation of a hysteresis loop by Martin Holters and Udo Zolzer, using the Jiles-Atherton magnetisation model can be found at http://dafx16.vutbr.cz/dafxpapers/08-DAFx-16_paper_10-PN.pdf. They use the following differential equation to describe magnetisation 'M' as a function of magnetic field 'H':

$$\frac{dM}{dH} = \frac{(1-c)\delta_M(M_{an} - M)}{(1-c)\delta k - \alpha(M_{an} - M)} + c\frac{dM_{an}}{dH}$$

where M_{an} is the anisotropic magnetisation given by:

$$M_{an} = M_s L\Big(\frac{H + \alpha M}{a}\Big)$$

where M_s is the magnetisation saturation, and L is the Langevin function:

$$L(x) = \coth(x) - \frac{1}{x}$$

$$L'(x) = \frac{1}{x^2} - \coth^2(x) + 1$$

$$L''(x) = 2\coth(x) \cdot (\coth^2(x) - 1) - \frac{2}{x^3}$$

Let $Q(t) = \frac{H+\alpha M}{a}$

Differentiating, we get:

$$\frac{dM}{dt} = \frac{(1-c)\delta_M(M_s L(Q) - M)}{(1-c)\delta k - \alpha(M_s L(Q) - M)}\frac{dH}{dt} + c\frac{M_s}{a}\Big(\frac{dH}{dt} + \alpha\frac{dM}{dt}\Big)L'(Q)$$

$$\frac{d^2M}{dt^2} = \frac{(1-c)\delta_M(M_sL(Q)-M)}{(1-c)\delta k-\alpha(M_sL(Q)-M)}\frac{d^2H}{dt^2}+\frac{(1-c)\delta_M(M_sL'(Q)-\dot{M})}{(1-c)\delta k-\alpha(M_sL(Q)-M)}\frac{dH}{dt}+\frac{(1-c)\delta_M(M_sL(Q)-M)(-\alpha(\ldots}{((1-c)\delta k-\alpha(M_sL(Q)\ldots}$$

Simplify:

$$\frac{d^2M}{dt^2} = \frac{\frac{(1-c)\delta_M(M_sL(Q)-M)}{(1-c)\delta k-\alpha(M_sL(Q)-M)}\frac{d^2H}{dt^2}+\frac{(1-c)\delta_M(M_sL'(Q)-\dot{M})}{(1-c)\delta k-\alpha(M_sL(Q)-M)}\frac{dH}{dt}+\frac{(1-c)\delta_M(M_sL(Q)-M)(-\alpha(M_sL'(Q)-\dot{M}))}{((1-c)\delta k-\alpha(M_sL(Q)-M))^2}\frac{dH}{dt}+c\frac{M_s}{a}\Big(\ldots}{1-c\alpha\frac{M_s}{a}L'(Q)}$$

$$\frac{dM}{dt} = \frac{\frac{(1-c)\delta_M(M_sL(Q)-M)}{(1-c)\delta k-\alpha(M_sL(Q)-M)}\frac{dH}{dt}+c\frac{M_s}{a}\frac{dH}{dt}L'(Q)}{1-c\alpha\frac{M_s}{a}L'(Q)} = f(t,M,\vec{u})$$

where $\vec{u} = \begin{bmatrix} H \\ \dot{H} \\ \ddot{H} \end{bmatrix}$

Using trapezoidal rule:

$$\dot{\hat{H}}(n) = 2\frac{\hat{H}(n)-\hat{H}(n-1)}{T} - \dot{\hat{H}}(n-1)$$

and similar for $\ddot{\hat{H}}$. Now, using the semi-implicit trapezoidal rule [Yeh]:

$$\hat{M}(n) = \hat{M}(n-1) + \frac{T}{2}\frac{f[n,\hat{M}(n-1),\vec{u}(n)]+f[n-1,\hat{M}(n-1),\vec{u}(n-1)]}{1-\frac{T}{2}\ddot{\hat{M}}(n-1)}$$