Continuous Time Equations for Analog Tape Modeling

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Record head

For an instantaneous current I, the magnetic field output of the record head is given as a function of distance along the tape 'x', and depth into the tape 'y' (Karlqvist medium field approximation) [Bertram, page 60]:

$$H_x(x,y) = \frac{1}{\pi} H_0 \left(\tan^{-1}\left(\frac{(g/2)+x}{y}\right) + \tan^{-1}\left(\frac{(g/2)-x}{y}\right)\right)$$
$$H_y(x,y) = \frac{1}{2\pi} H_0 \ln\left(\frac{((g/2)-x)^2+y^2}{((g/2)+x)^2+y^2}\right)$$

where g = head gap, and H_0 = deep gap field, given by:

$$H_0 = \frac{NIE}{g}$$

where N = number of turns coils of wire around the head, and E = head efficiency given by:

$$E = \frac{1}{1 + \frac{lA_g}{\mu_r g} \int_{core} \frac{d\vec{l}}{A(l)}}$$

where A_g is the gap area, μ_r is the core permeability relative to free space (μ_0), g if the gap width, and $A(l)$ is the cross-sectional area of the core as a function of length.

Tape Magnetisation

Deadzone

For low current, the field is insufficient to create a change in magnetisation. For high current the field saturates. The effective field magnetising the tape H_h can be described as follows:

$$H_h = \begin{cases} 0 & H \leq S^* H_c \\ H & H > S^* H_c \end{cases}$$

where S^* = hysteresis loop squareness, and H_c = coercivity.

Hysteresis

The magnetostatic field recorded to magnetic tape can be described using a hysteresis loop. A circuit simulation of a hysteresis loop by Martin Holters and Udo Zolzer, using the Jiles-Atherton magnetisation model can be found at http://dafx16.vutbr.cz/dafxpapers/08-DAFx-16_paper_10-PN.pdf. They use the following differential equation to describe magnetisation 'M' as a function of magnetic field 'H':

$$\frac{dM}{dH} = \frac{(1-c)\delta_M (M_{an} - M)}{(1-c)\delta k - \alpha(M_{an} - M)} + c\frac{dM_{an}}{dH}$$

where M_{an} is the anisotropic magnetisation given by:

$$M_{an} = M_s L\Big(\frac{H + \alpha M}{a}\Big)$$

where M_s is the magnetisation saturation, and L is the Langevin function.

Playback head

Ideal playback voltage

The ideal playback voltage as a function of tape magnetisation is given by [Bertram, page 121]:

$$V(x) = NWEv\mu_0 \int_{-\infty}^{\infty} dx' \int_{-\delta/2}^{\delta/2} dy' \vec{h}(x' + x, y') \cdot \frac{\vec{M}(x', y')}{dx}$$

where N = number of turns of wire, W = with of the playhead, E = playhead efficiency, v = tape speed. Note that $V(x) = V(vt)$ for constant v. $\vec{h}(x, y)$ is defined as:

$$\vec{h}(x,y) \equiv \frac{\vec{H}(x,y)}{NIE}$$

where $\vec{H}(x,y)$ is the same as for the record head.

Loss effects

There are several frequency-dependent loss effects associated with playback, described as follows [Kadis, page 126]:

$$V(x) = V_0(x)[e^{-kd}]\left[\frac{1-e^{-k\delta}}{k\delta}\right]\left[\frac{\sin(kg/2)}{kg/2}\right]$$

where k = wave number.

Spacing Loss

$$g_s = e^{-kd}$$

where d is the distance between the tape and the playhead.

Gap Loss

$$g_g = \frac{\sin(kg/2)}{kg/2}$$

where g is the gap with of the play head.

Thickness Loss

$$g_t = \frac{1-e^{-k\delta}}{k\delta}$$

where δ is the thickness of the tape.

Conclusion

If each of these components is digitized, a digital physical model of the analog tape machine can be constructed.